

# **O uso da matemática e computação na resolução de quebra-cabeças**

**Aline Leão**

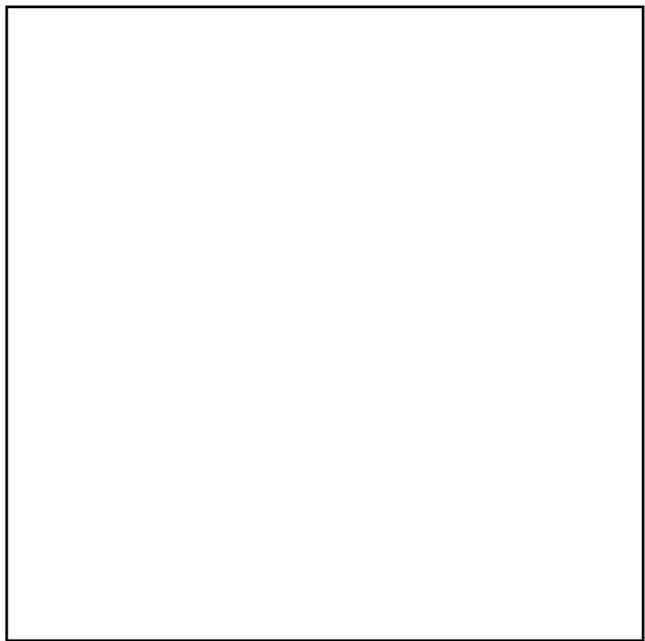
**Universidade Estadual de Londrina**

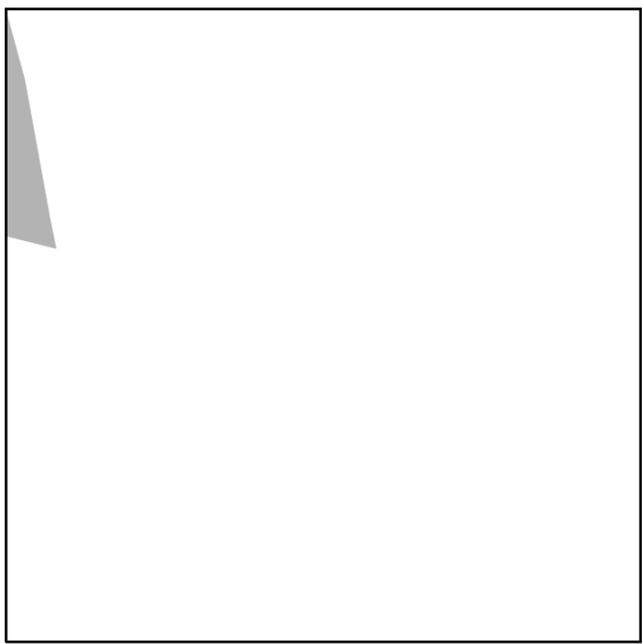
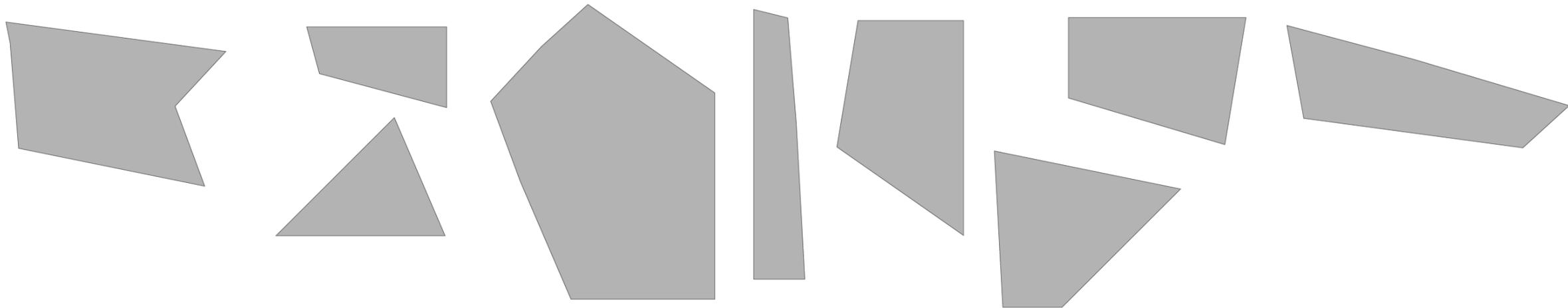
**Departamento de Matemática**

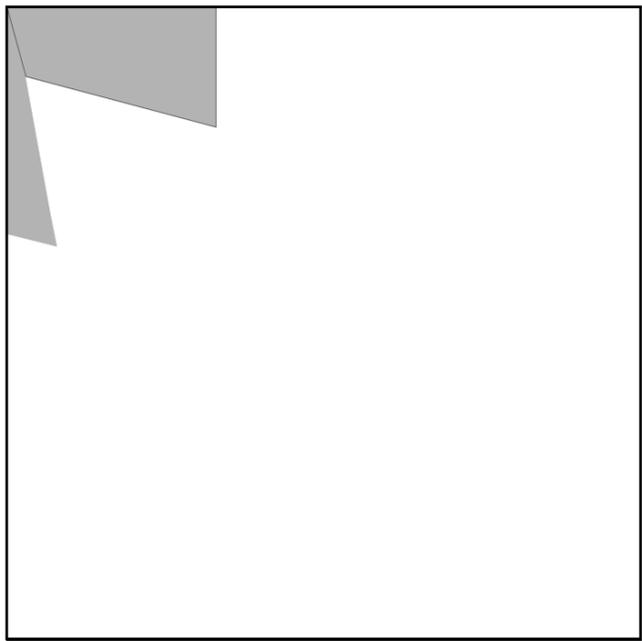
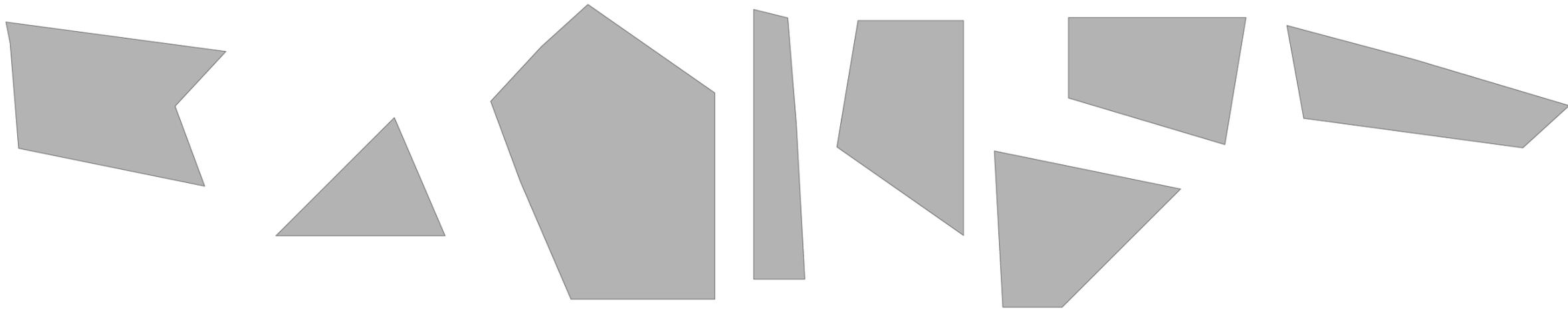
**aasleao@uel.br**

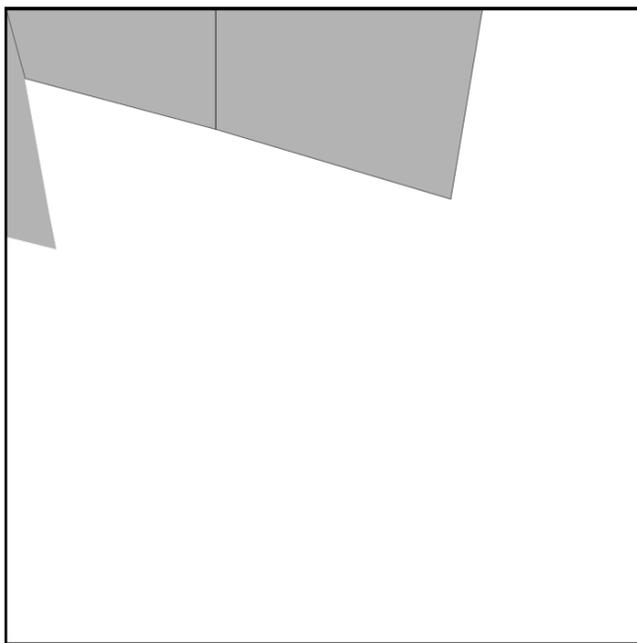
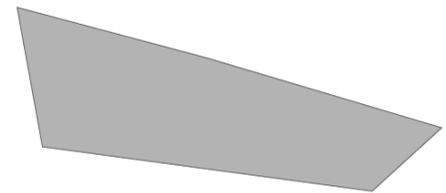
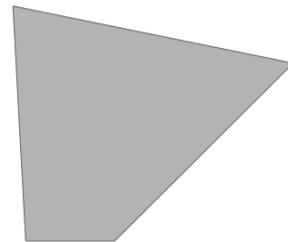
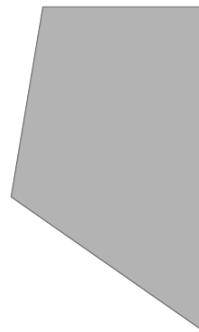
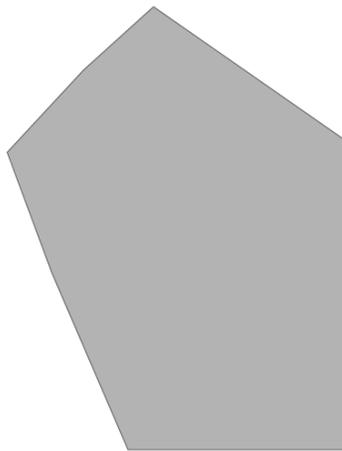
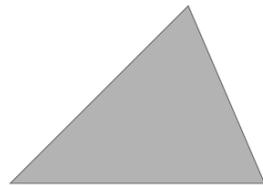
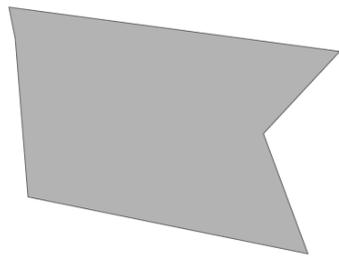


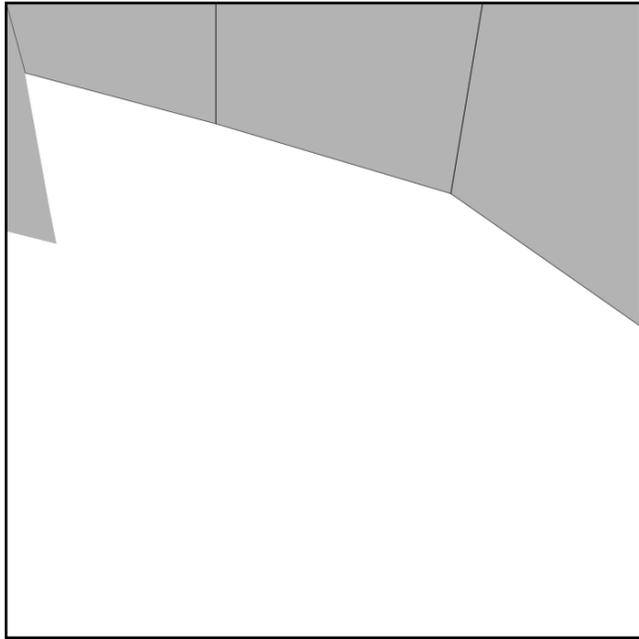
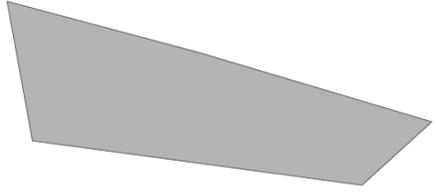
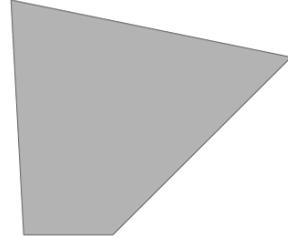
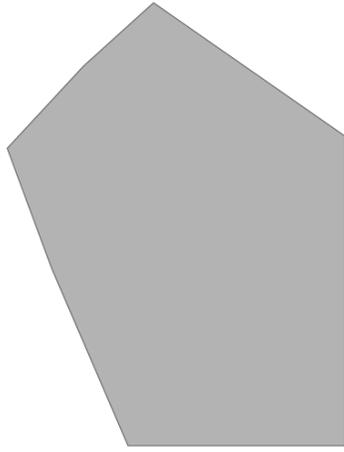
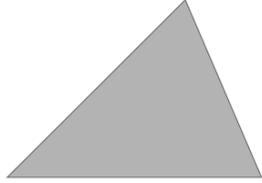
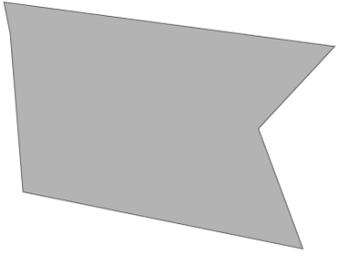
- Aplicações
- Definição
- Ferramentas geométricas
- Modelagem matemática
- Direções de pesquisas futuras

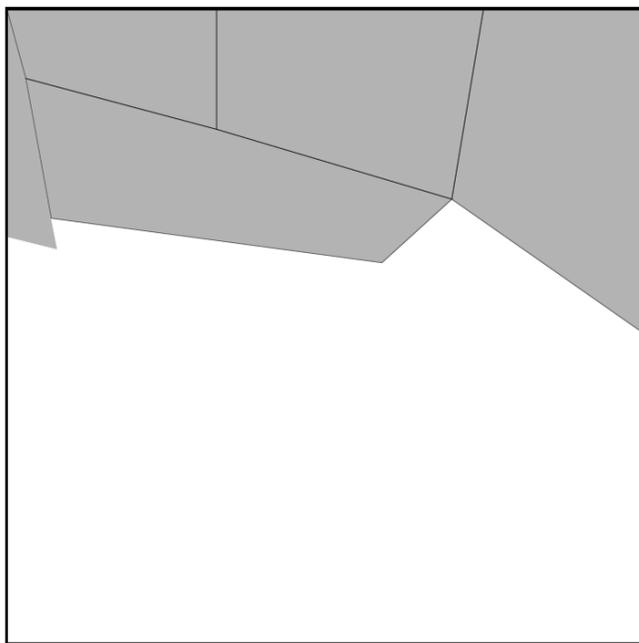
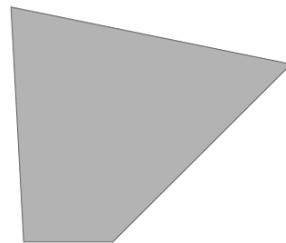
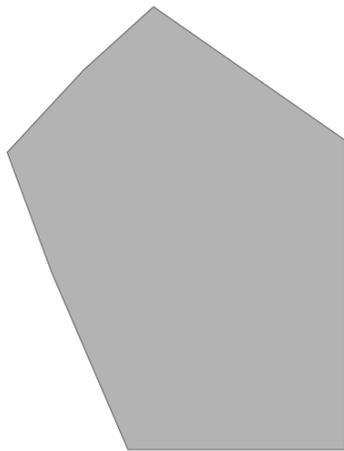
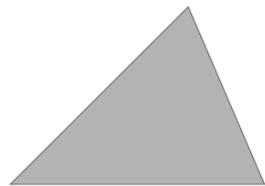
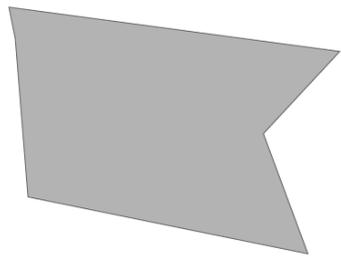


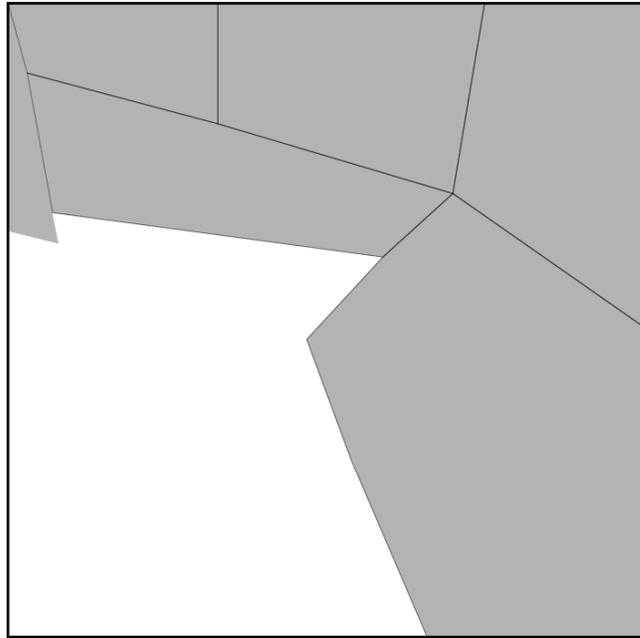
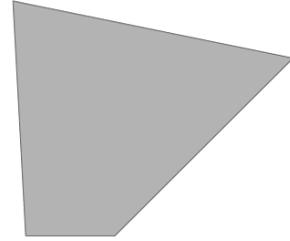
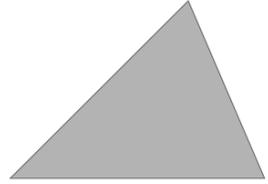
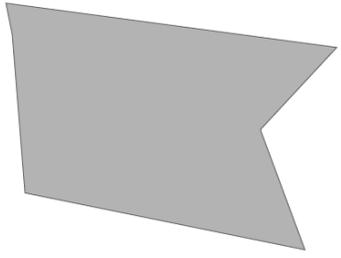


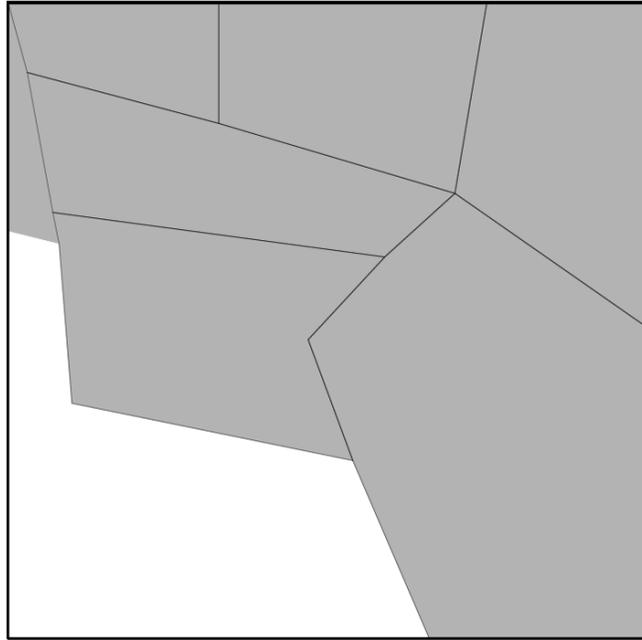
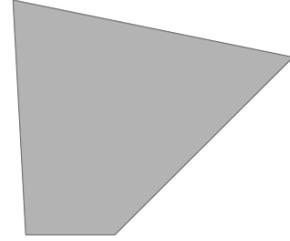
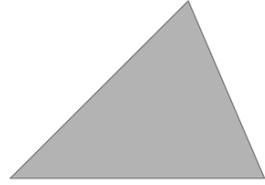


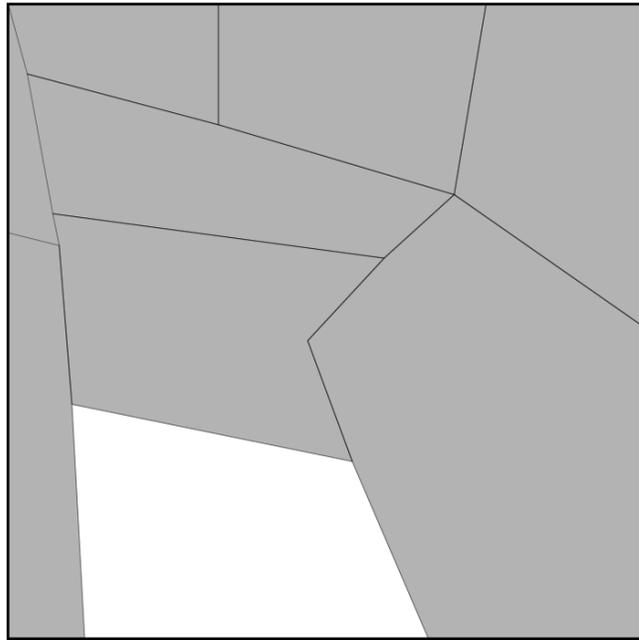
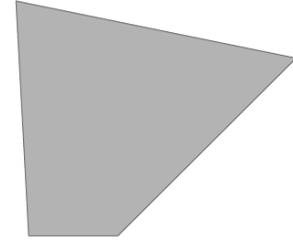
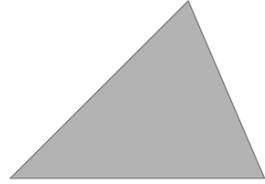


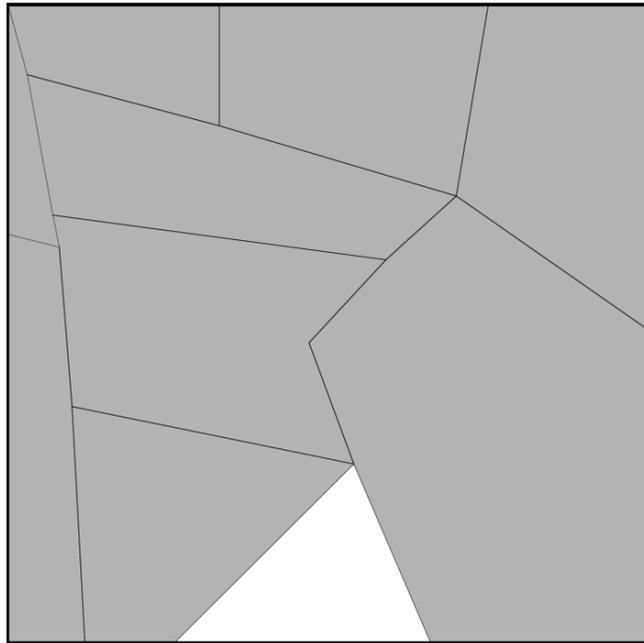
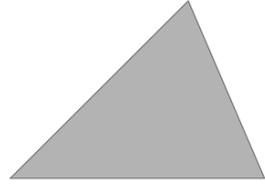


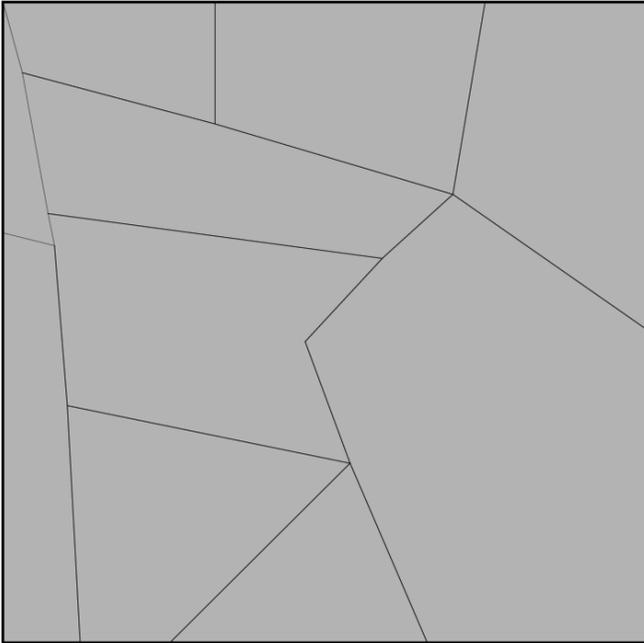




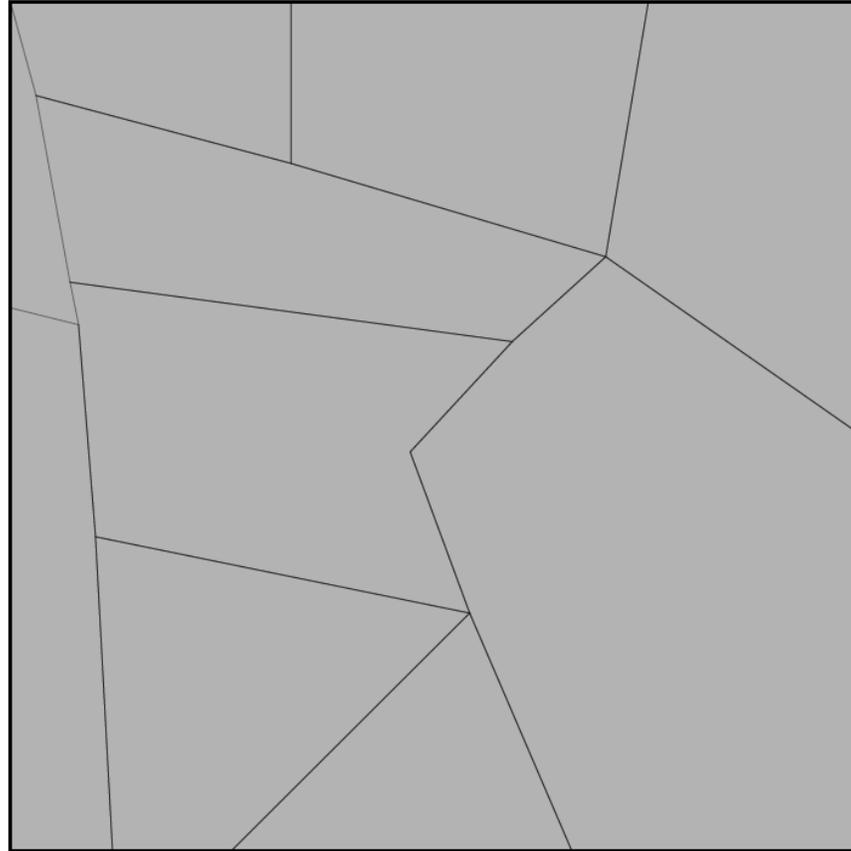








## Problemas de corte e empacotamento de peças irregulares

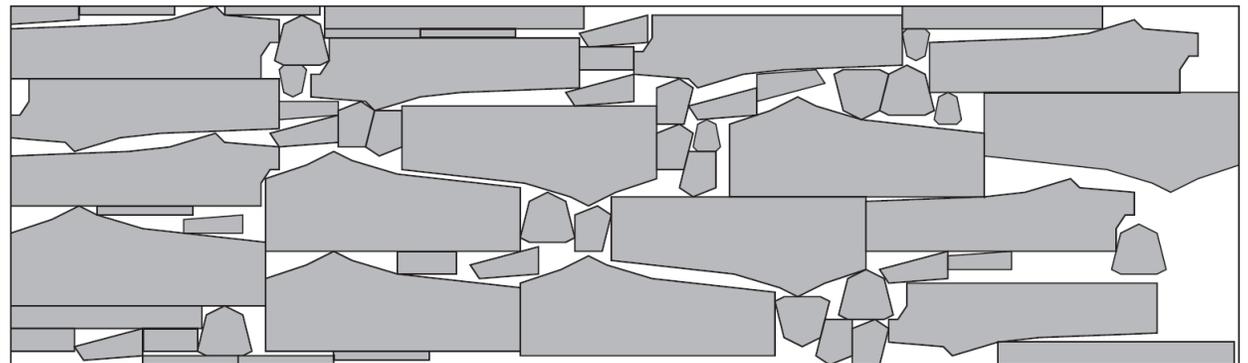


## Aplicações industriais

- Roupas e sapatos



retirado de [www.homemnamoda.com.br](http://www.homemnamoda.com.br)

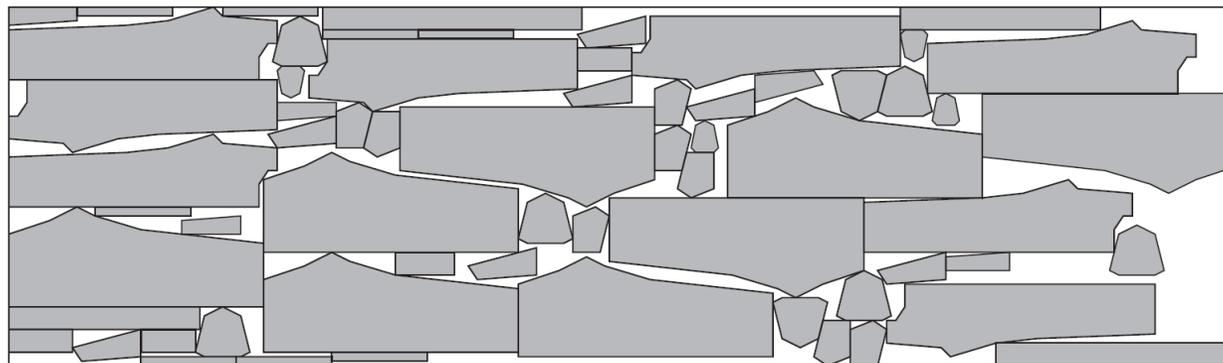


## Aplicações industriais

- Roupas e sapatos
- Móveis



retirado de [www.homemnamoda.com.br](http://www.homemnamoda.com.br)



## Aplicações industriais

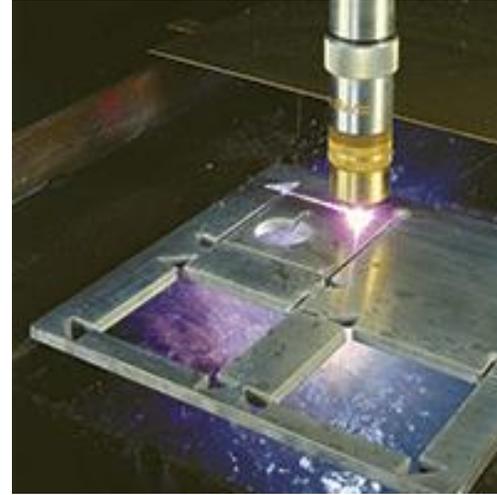
- Roupas e sapatos
- Móveis
- Vidro



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## Aplicações industriais

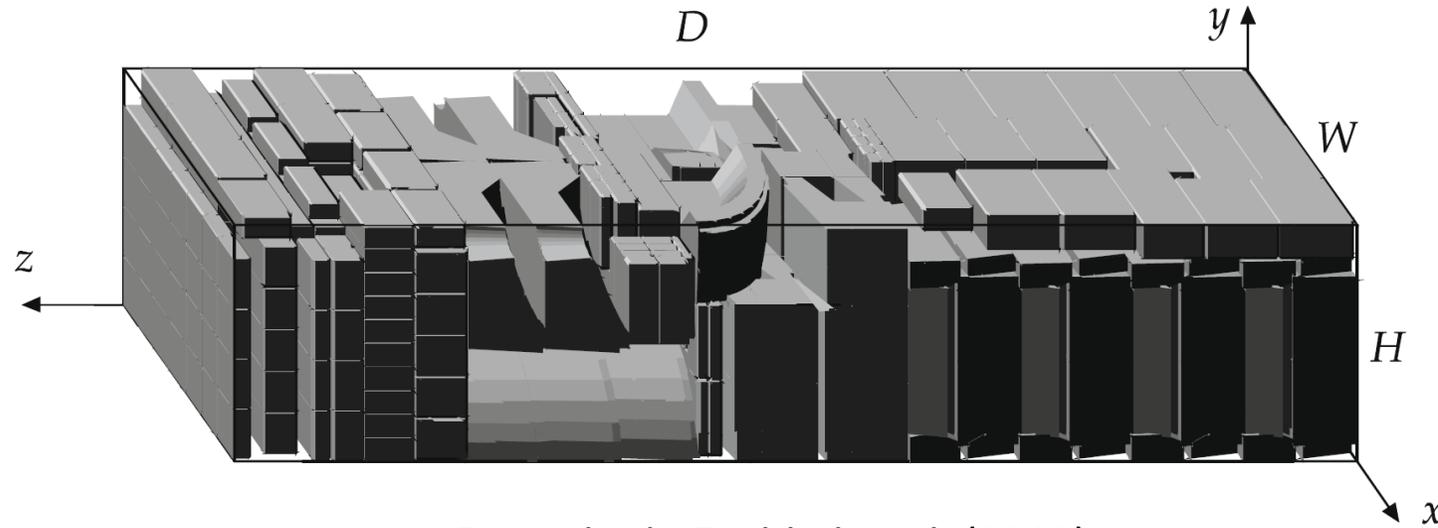
- Roupas e sapatos
- Móveis
- Vidro
- Automobilísticas
- Metalúrgicas



retirado de [www.sigmanest.com](http://www.sigmanest.com)

## Aplicações industriais

- Roupas e sapatos
- Móveis
- Vidro
- Automobilísticas
- Metalúrgicas
- Carregamento de contêiner



Retirado de Egeblad et al. (2010)

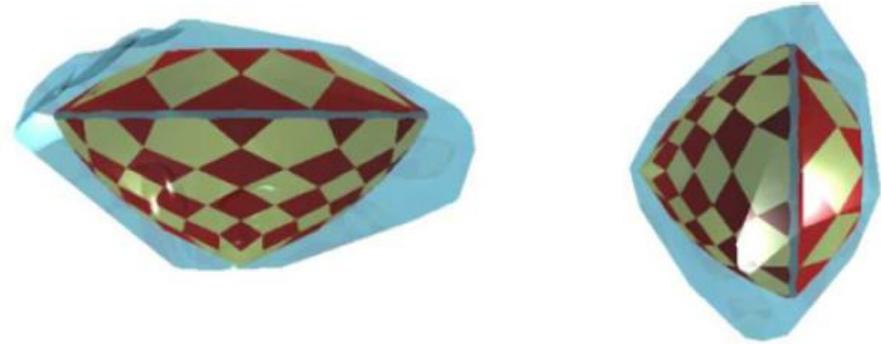
## Aplicações industriais

- Roupas e sapatos
- Móveis
- Vidro
- Automobilísticas
- Metalúrgicas
- Carregamento de contêiner
- Corte de espuma



## Aplicações industriais

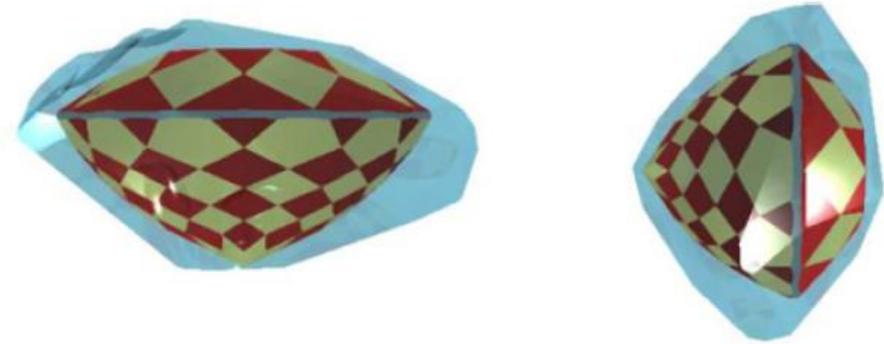
- Roupas e sapatos
- Móveis
- Vidro
- Automobilísticas
- Metalúrgicas
- Carregamento de contêiner
- Corte de espuma
- Lapidação de pedras preciosas



Retirado de Winterfeld (2008)

## Aplicações industriais

- Roupas e sapatos
- Móveis
- Vidro
- Automobilísticas
- Metalúrgicas
- Carregamento de contêiner
- Corte de espuma
- Lapidação de pedras preciosas
- Cerâmica
- Impressoras 3D

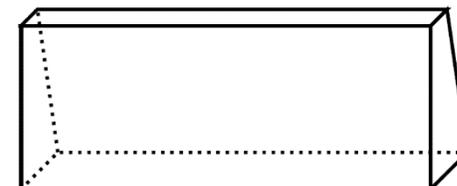
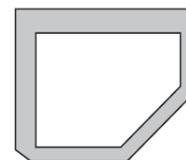
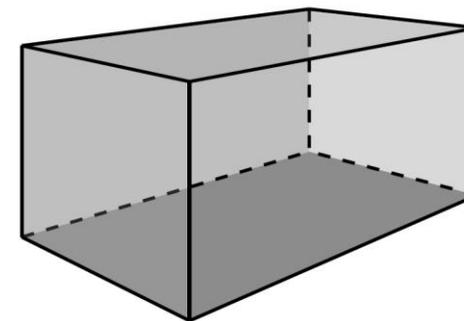
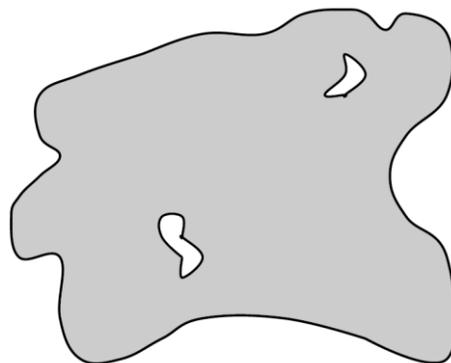


Retirado de Winterfeld (2008)

## Definição

- Um objeto ou um conjunto de objetos:  $\mathbf{P}^0$  ou  $\mathbf{P}^{0b}$ ,  $b = 1, \dots, n$
- Conjunto de peças:  $\mathbf{P}^i$ ,  $i = 1, \dots, m$

- Descritos por vértices, arcos ou união de figuras geométricas primitivas



## Definição

### Posicionamento das peças nos objetos

- Dada uma peça representada por  $\mathbf{P}$  e um vetor  $\mathbf{v}$ , o conjunto  $\mathbf{P} \oplus \mathbf{v} = \{\mathbf{p} + \mathbf{v} : \mathbf{p} \in \mathbf{P}\}$  define o a peça  $\mathbf{P}$  transladada por  $\mathbf{v}$ , onde  $\mathbf{p}$  é um ponto em  $\mathbf{P}$  com coordenadas  $(x^{\mathbf{p}}, y^{\mathbf{p}})$ .
- $\mathbf{P}(\theta) = \{(x^{\mathbf{p}} \cos(\theta) - y^{\mathbf{p}} \sin(\theta), x^{\mathbf{p}} \sin(\theta) + y^{\mathbf{p}} \cos(\theta)) : \mathbf{p} = (x^{\mathbf{p}}, y^{\mathbf{p}}) \in \mathbf{P}\}, \theta \in \Theta$ .

## Definição

### Restrições

- 1) Se um peça é empacotada, ela deve estar dentro do objeto:  $\mathbf{P}^i(\theta_i) \oplus \mathbf{v}^i \subseteq \mathbf{P}^{0b}$ , para algum  $b$ .
- 2) Se as peças  $i$  e  $j$  são empacotadas na mesma placa, elas não podem se sobrepor:  $int(\mathbf{P}^i(\theta_i) \oplus \mathbf{v}^i) \cap int(\mathbf{P}^j(\theta_j) \oplus \mathbf{v}^j) = \emptyset$ ,  $int(\mathbf{P})$  é o interior da peça.

## Definição

minimize  $waste$  (1)

subject to:

$$(\mathbf{P}^i(\theta_i) \oplus \mathbf{v}^i) \subset \mathbf{P}^{00} \vee \dots \vee (\mathbf{P}^i(\theta_i) \oplus \mathbf{v}^i) \subset \mathbf{P}^{0n} \quad i = 1, \dots, m \quad (2)$$

$$(\mathbf{P}^i(\theta_i) \oplus \mathbf{v}^i) \cap (\mathbf{P}^j(\theta_j) \oplus \mathbf{v}^j) \subset \mathbf{P}^{0b} = \emptyset \quad \forall i, j \text{ placed in } \mathbf{P}^{0b}, b = 1, \dots, n \quad (3)$$

$$\theta_i \in \Theta, \quad \mathbf{v}^i \in \mathbb{R}^2 \quad i = 1, \dots, m. \quad (4)$$

## Ferramentas geométricas

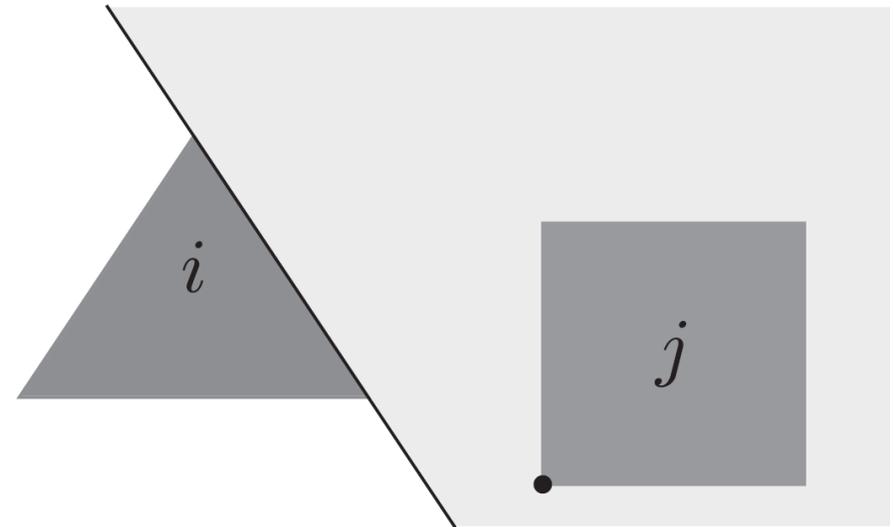
### Trigonometria direta: função D

$$D_{abP} = (a_x - b_x)(a_y - y) - (a_y - b_y)(a_x - x)$$

$D_{abP} = 0$  se  $P$  está sobre a reta  $AB$

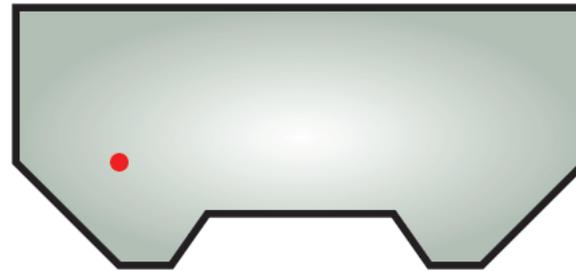
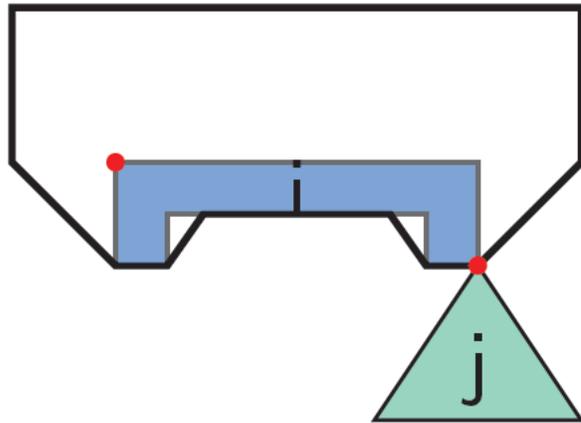
$D_{abP} > 0$  se  $P$  está à esquerda a reta  $AB$

$D_{abP} < 0$  se  $P$  está à direita da reta  $AB$



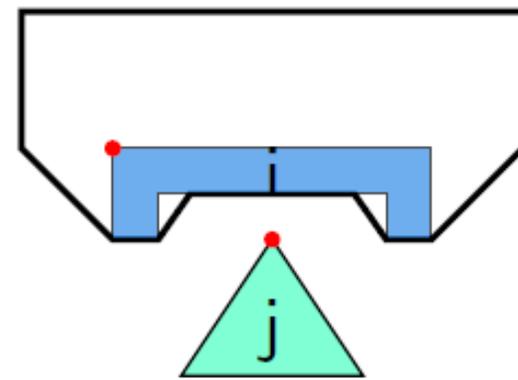
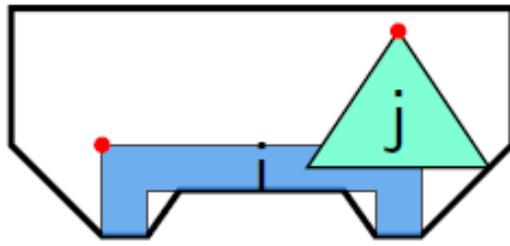
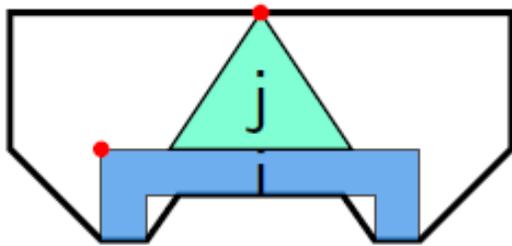
# Ferramentas geométricas

*No-fit polygon*



# Ferramentas geométricas

*No-fit polygon*

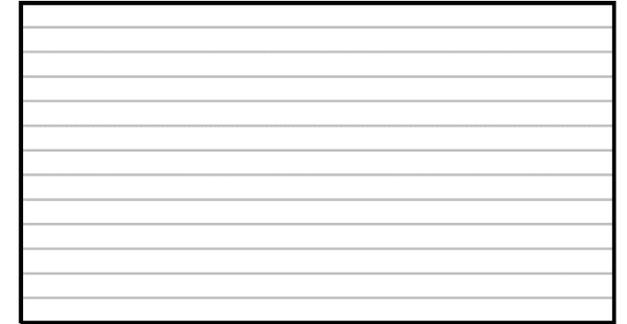
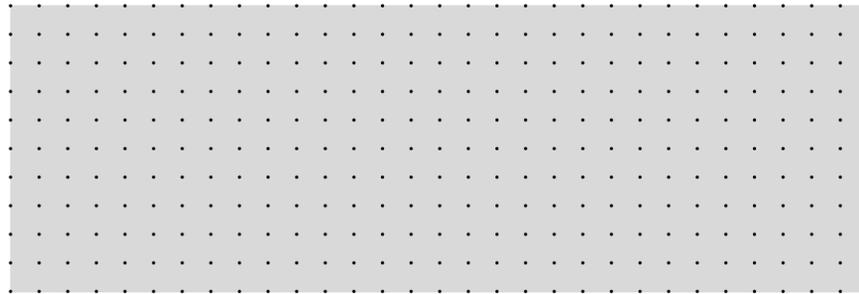
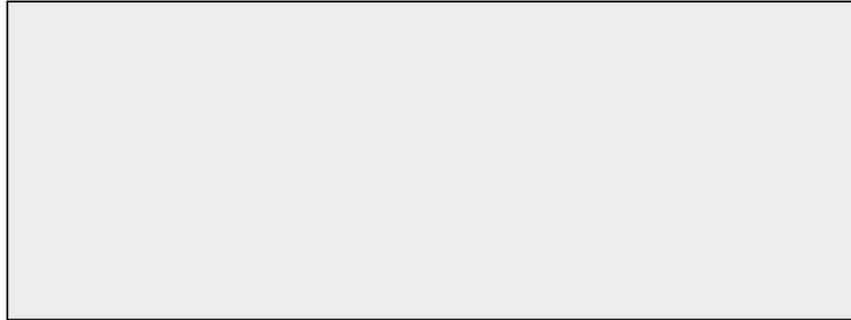


# Ferramentas geométricas

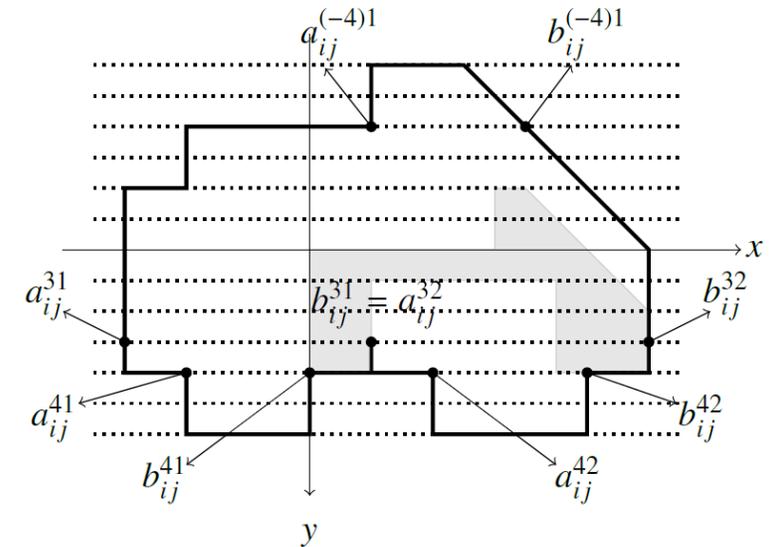
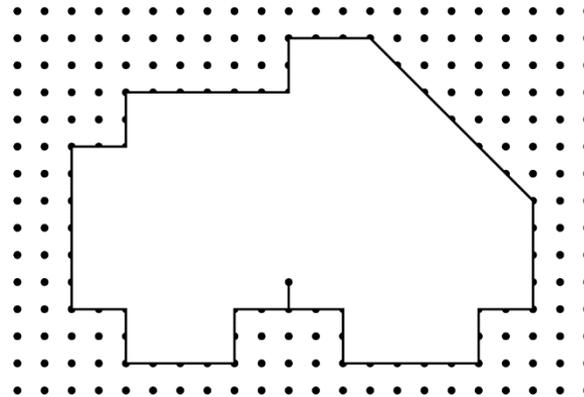
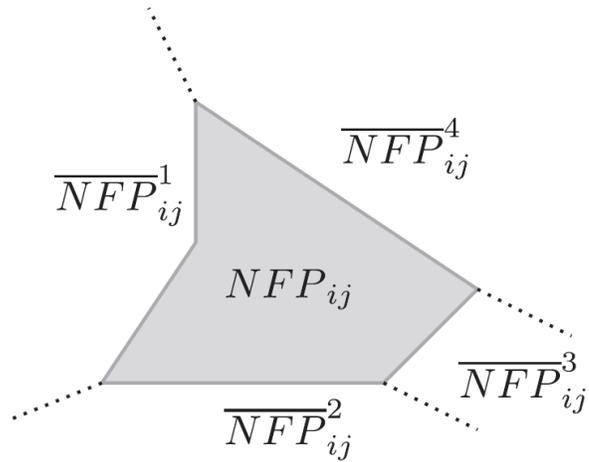
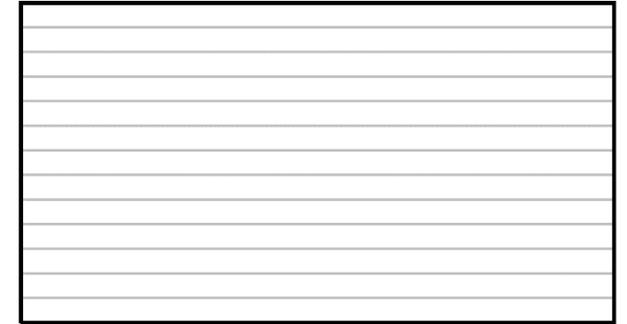
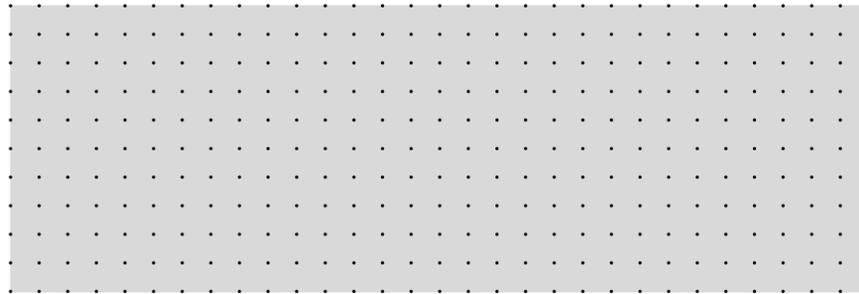
## *Inner-fit polygon*



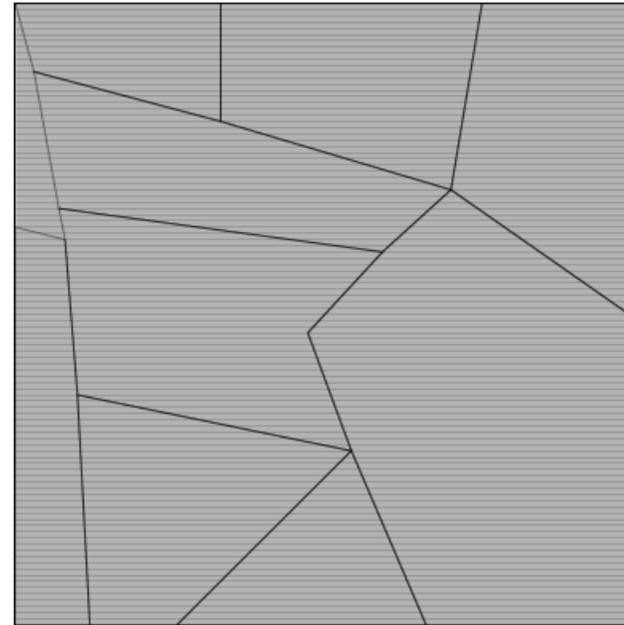
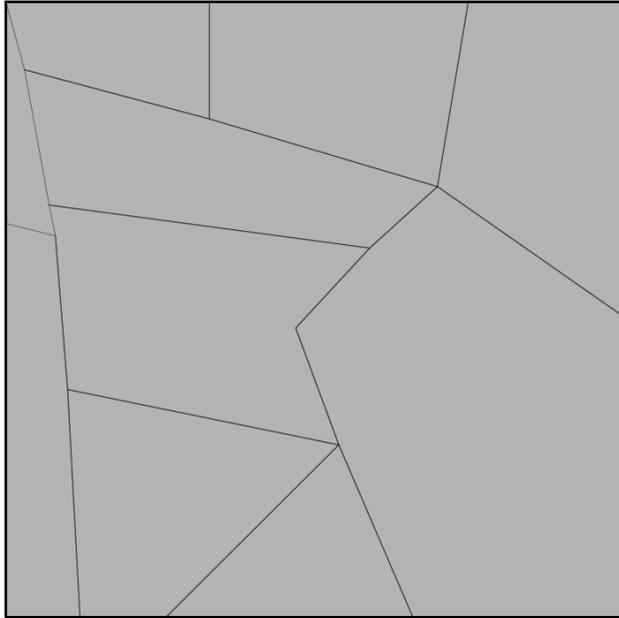
# Modelagem matemática: posicionamento semicontínuo



# Modelagem matemática: posicionamento semicontínuo

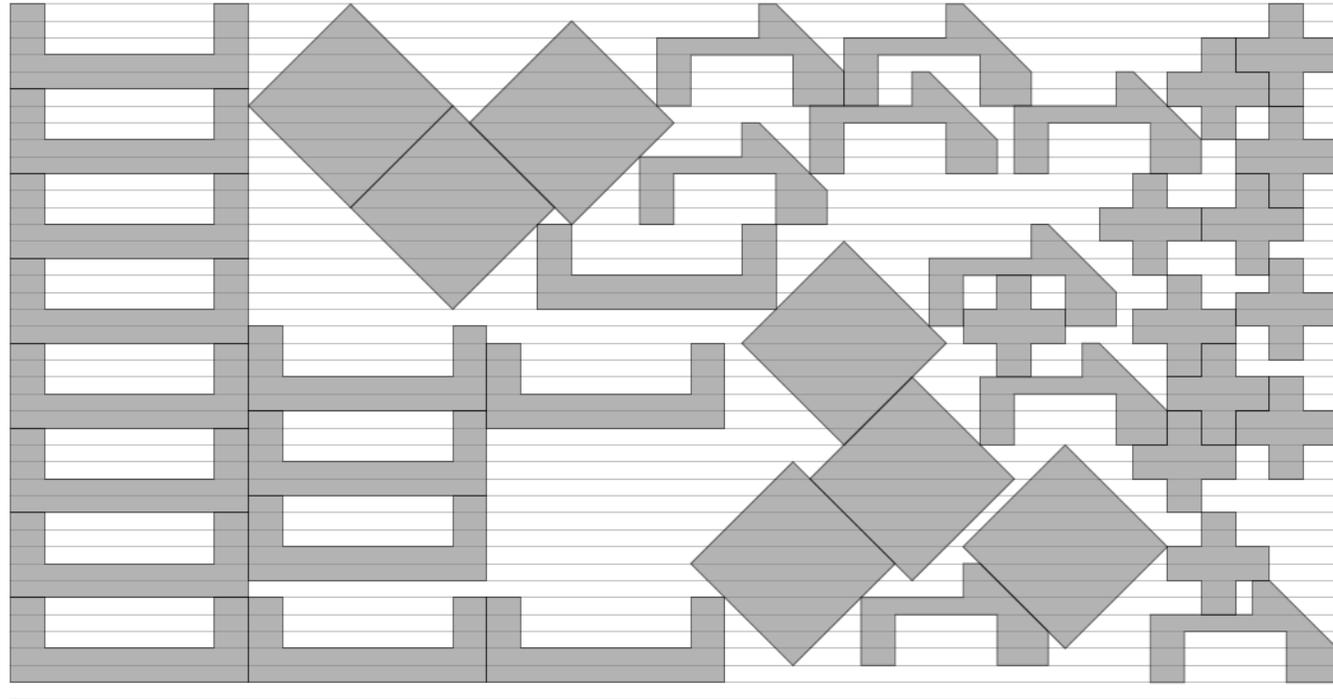


# Modelagem matemática: posicionamento semicontínuo



dighe2: Width: 100, z = 100.

## Modelagem matemática: posicionamento semicontínuo



Largura 41, comprimento 78, 43 peças

813864 restrições e 3143 variáveis binárias.

# Modelagem matemática: posicionamento semicontínuo

Restrições de não sobreposição:

- $$x_i \leq x_j - b_{ij}^{(s_j - s_i)c} + \gamma_{ij}^c M + (1 - \delta_i^{s_i})M + (1 - \delta_j^{s_j})M$$

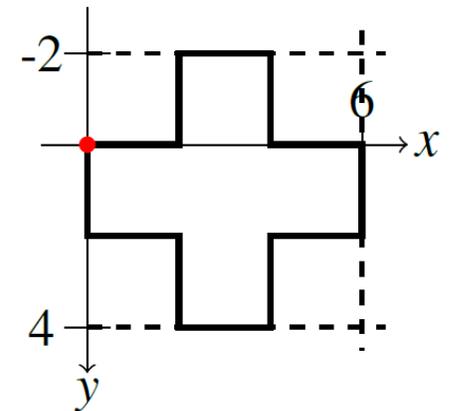
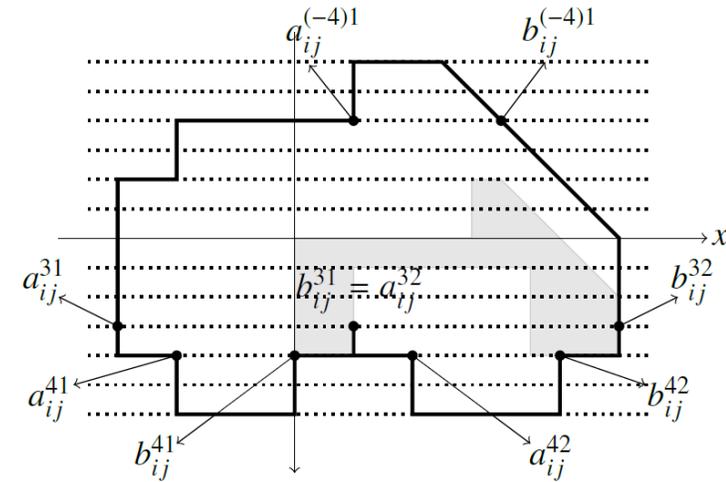
$$1 \leq i < j \leq m, c = 1, \dots, C_{ij}^{s_j - s_i}, -t_i^{\min} \leq s_i \leq S - t_i^{\max},$$

$$-t_j^{\min} \leq s_j \leq S - t_j^{\max}, n_{ij}^{\min} \leq s_j - s_i \leq n_{ij}^{\max}, \quad (5)$$

- $$x_i \geq x_j - a_{ij}^{(s_j - s_i)c} - (1 - \gamma_{ij}^c)M - (1 - \delta_i^{s_i})M - (1 - \delta_j^{s_j})M$$

$$1 \leq i < j \leq m, c = 1, \dots, C_{ij}^{s_j - s_i}, -t_i^{\min} \leq s_i \leq S - t_i^{\max},$$

$$-t_j^{\min} \leq s_j \leq S - t_j^{\max}, n_{ij}^{\min} \leq s_j - s_i \leq n_{ij}^{\max}, \quad (6)$$



Piece  $j$ :  $t^{\min} = -2$ ;  $t^{\max} = 4$ ;  
 $r^{\min} = 0$ ;  $r^{\max} = 6$ .

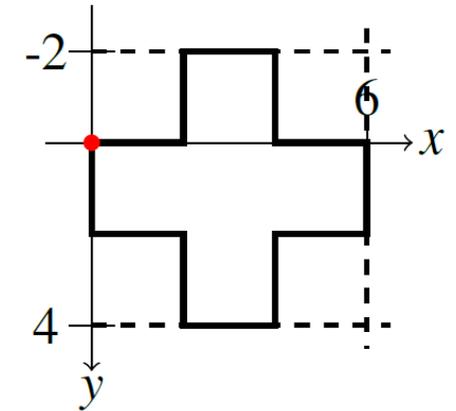
## Modelagem matemática: posicionamento semicontínuo

Restrições que garantem que as peças estão dentro do objeto:

$$\bullet L \geq x_i + r_i^{max}, 1 \leq i \leq m \quad (7)$$

$$\bullet x_i \geq -r_i^{min}, 1 \leq i \leq m \quad (8)$$

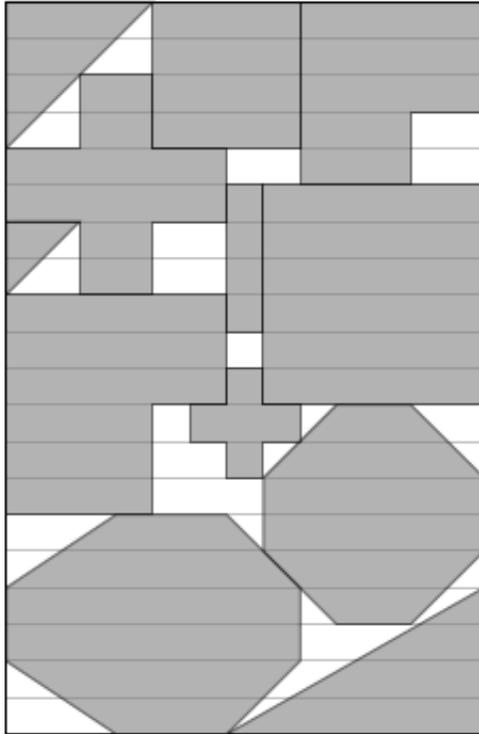
$$\bullet \sum_{s=t_i^{min}}^{s-t_i^{max}} \delta_i^s = 1, 1 \leq i \leq m \quad (9)$$



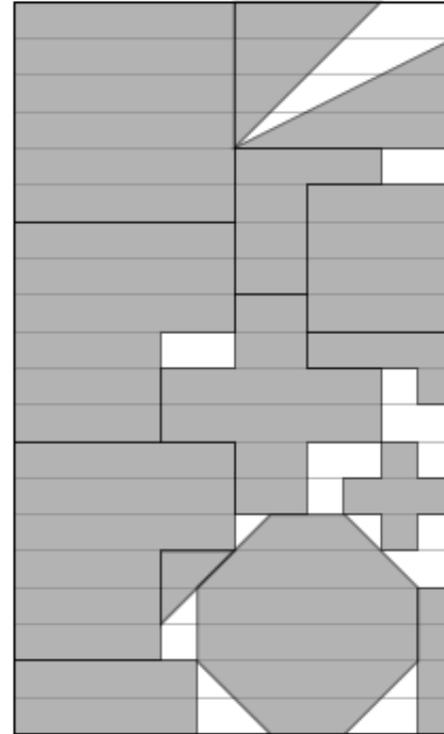
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 $r^{min} = 0$ ;  $r^{max} = 6$ .

# Modelagem matemática: posicionamento semicontínuo

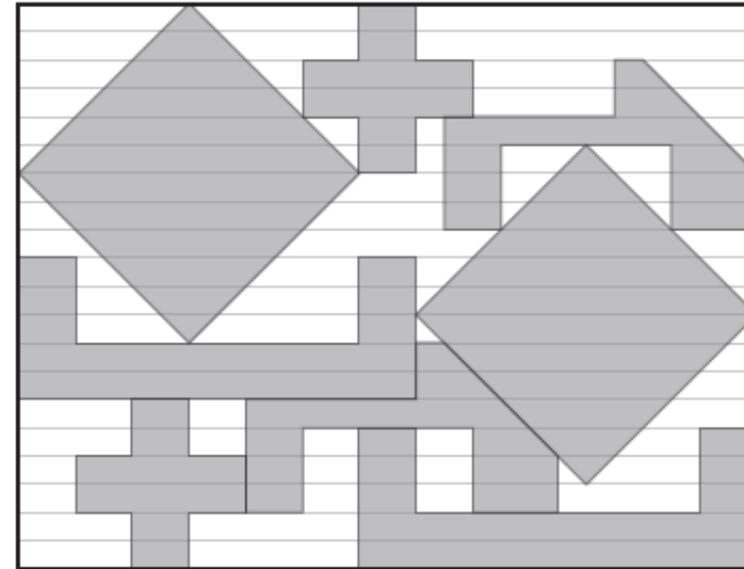
Padrões de corte:



J1-12-20-4: Width = 20, z = 13.



J1-14-20-0: Width = 20, z = 12.



shapes8: Width: 20, z = 26.

## Direções de pesquisas futuras

- Definir modelos matemáticos melhores
- Melhorar métodos de solução
- Rotação contínua de peças
- Objetos irregulares
- Classificação da qualidade de regiões
- Problemas tridimensionais irregulares

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# **O uso da matemática e computação na resolução de quebra-cabeças**

**Aline Leão**

**Universidade Estadual de Londrina**

**Departamento de Matemática**

**aasleao@uel.br**

